Problem 4.7

The position and velocity of a fish is:

$$\vec{r}_1(t) = \left(10\hat{i} - 4\hat{j}\right)\text{m} \qquad \text{and} \qquad \vec{v}_1(t) = \left(4\hat{i} + 1\hat{j}\right)\text{m/s}$$

After 20.0 seconds, it's velocity is:

$$\vec{v}_2(t) = \left(20\hat{i} - 5\hat{j}\right)\text{m/s}$$

a.) What are the acceleration components:

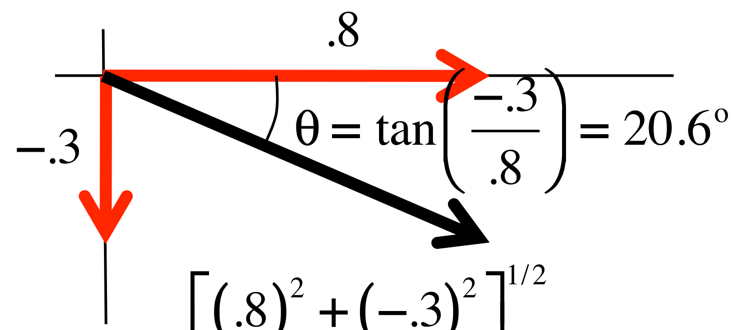
$$a(t) = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$$

$$= \frac{\left[\left(20\hat{i} - 5.00\hat{j}\right) - \left(4\hat{i} + 1\hat{j}\right)\right]\text{m/s}}{(20\ \text{s})}$$

$$= \left(.8\hat{i} - .3\hat{j}\right)\text{m/s}^2$$

So $a_x = .8\ \text{m/s}^2$ and $a_y = -.3\ \text{m/s}^2$

b.) This is asking us to convert to polar notation. As I usually do when doing something like this, I've created a graph (see sketch). (I've actually determined more info than needed—such is life.)



$$\left[(.8)^2+(-.3)^2\right]^{1/2} = .854$$

$$\begin{aligned} a(t) &= \left(.8\hat{i} - .3\hat{j}\right)\text{m/s}^2 \\ &= \left[(.8)^2+(-.3)^2\right]^{1/2} \tan\left(\frac{-.3}{.8}\right) \\ &= \left(.854 \text{ m/s}^2\right)\measuredangle -20.6^{\circ} \end{aligned}$$

c.) After 25 seconds:

Knowing the constant acceleration, we are dealing with kinematics here. As we also know the initial position, initial velocity and time, we can write::

$$\begin{aligned} x_2 &= x_1 + v_{1,x}\Delta t + \frac{1}{2}a_x(\Delta t)^2 \\ &= (10 \text{ m}) + (4 \text{ m/s})(25 \text{ s}) + \frac{1}{2}\left(.8 \text{ m/s}^2\right)(25 \text{ s})^2 \\ &= 360 \text{ m} \end{aligned}$$

and:

$$
\begin{aligned}
y_2 &= y_1 + v_{1,y}\Delta t + \frac{1}{2}a_y(\Delta t)^2 \\
&= (-4 \text{ m}) + (1 \text{ m/s})(25 \text{ s}) + \frac{1}{2}(-.3 \text{ m/s}^2)(25 \text{ s})^2 \\
&= -72.7 \text{ m}
\end{aligned}
$$

or:

$$
\begin{aligned}
\vec{r}_2 &= \quad x_2 \quad \hat{i} + \quad y_2 \quad \hat{j} \\
&= (360 \text{ m})\hat{i} + (72.7 \text{ m})(-\hat{j}) \\
&= (360 \text{ m})\hat{i} - (72.7 \text{ m})\hat{j} \qquad \text{(where this is the more normal notation)}
\end{aligned}
$$

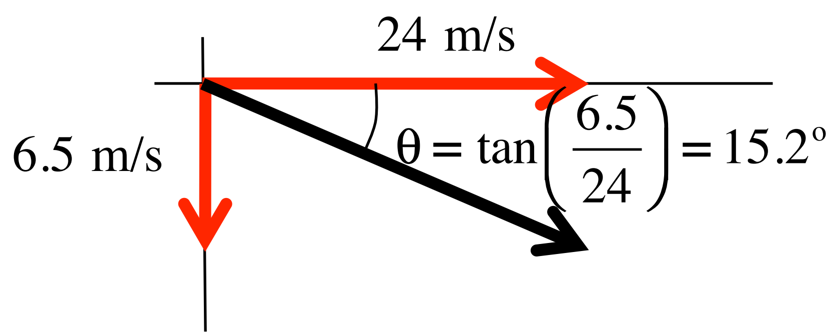
The book directions requires us to know the net velocity at time 2 seconds. To get those components, we write

$$v_{2,x} = v_{1,x} + a_x \Delta t$$
$$= (4\ \text{m/s}) + (.8\ \text{m/s}^2)(25\ \text{s})$$
$$= 24\ \text{m/s}$$

and:

$$v_{2,y} = v_{1,y} + a_y \Delta t$$
$$= (1\ \text{m/s}) + (-.3\ \text{m/s}^2)(25\ \text{s})$$
$$= -6.5\ \text{m/s}$$

so:



and:

$$\theta = \tan\left(\frac{-6.5}{24}\right)$$
$$= -15.2^{\circ}$$